Component-Wise Method Applied to Static and Dynamic Analysis of Reinforced Structures with Applications to Aerospace, Civil Engineering and Marine Constructions

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COMPONENT-WISE METHOD APPLIED TO STATIC AND DYNAMIC ANALYSIS OF REINFORCED STRUCTURES WITH APPLCATIONS TO AEROSPACE, CIVIL ENGINEERING AND MARINE CONSTRUCTIONS

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Abstract: In this work, an advanced formulation for the analysis of multi-component structures is presented. By employing the Carrera Unified Formulation (CUF), one-dimensional theories of structures are unified and written in a compact form by using fundamental nuclei. The principle of virtual displacement is then used to write the governing equations and the related finite element arrays. Classical one-dimensional shape functions are utilized to discretize the problem along the beam axis and to deal with complex geometries and loadings. Thanks to CUF, various one-dimensional beam theories are included within the same hierarchical finite element. Particular attention is paid to the Component-Wise (CW) approach. CW models are generated by developing beam theories based on Lagrange polynomial expansions of the generalized displacements. The enhanced capabilities of the present CW models when applied to the analysis of short beams, thin-walled structures and multi-component constructions are widely discussed.
Keywords: Multi-component structures, Component-wise approach, Carrera Unified Formulation, Finite Element Method

Most of the engineering structures(e.g., aircraft wings, laminated composite structures, etc.) are made of different components. Each of these components, generally, have each own geometry, scale, and mechanical behaviour. In the case of aircraft wing structures, for example, one-dimensional slender bodies such as spars and stringers undergo axial stress; panels are subjected to shear stress; whereas ribs have high in-plane stiffness and are used to carry loads. The choice of the correct modelling approach for each component is, essentially, part of the knowledge of the structural analyst.

Since a long time, within the framework of Finite Element Method (FEM),reinforced-shells such as aircraft structures are modelled by employing a combination of 1D/beam and 2D/plate-shell elements (see for example [1]). In this approach,it is often necessary to model stiffeners out of the plate/shell element plane. In this case, beam nodes are connected to theshell element nodes via rigid fictitious links. This methodologypresents some inconsistencies. The main problem is that the out-of-planewarping displacements in the stiffener section are neglected,and the beam torsional rigidity is not correctly predicted.Severalsolutions have been proposed in the literature to overcome this issue, such as [2]. However, the only solution to overcome the geometrical inconsistencies in the modelling of multi-component structures appears to be 3D finite elements, which, despite the availabilities of even cheaper computerpower, present large computational costs.

In this paper, an innovative approach to the analysis of multi-component structures is proposed. The methodology is based on the Carrera Unified Formulation (CUF), which states that, in the case of one-dimensional structural theories, the displacement field can be expressed as a generic expansion of arbitrary functions of the cross-section coordinates [3].

$$\mathbf{u}(x,y,z) = \mathbf{u}_\tau(y) F_\tau(x,z), \qquad \tau = 1,\ldots,M$$

Where $\mathbf{u}(x,y,z)$ is the 3D displacement field vector, $\mathbf{u}_\tau(y)$ is the vector of the generalized displacements, $F_\tau(x,z)$ is an arbitrary set of functions that determine the beam theory order and class, M is the number of expansion terms, and the subscript τ is a summation index.

By using CUF, the displacement-strain differential equations and the constitutive relations within the principle of virtual displacements, the finite element matricescan be developed in terms of fundamental nuclei. For example, by considering the virtual variation of the internal work, one has:

$$\delta L_{\text{int}} = \delta \mathbf{q}^{i\tau} \mathbf{K}^{ij\tau s} \mathbf{q}^{js}$$

Where $\mathbf{q}^{js}$ is the nodal unknowns vector and $\mathbf{K}^{ij\tau s}$ is the 3x3 fundamental nucleus of the stiffness matrix. The formal expression of the fundamental nucleus does not depend on the choice of $F_\tau(x,z)$ functions. The element stiffness matrix can be automatically obtained by expanding $\mathbf{K}^{ij\tau s}$ $K^{ij\tau s}$ versus the theory of structures indexes (τ and s) and the finite element indexes (i and j).

In this paper, particular attention is given to a particular class of CUF theories which is based on the use of Lagrange polynomials as $F_\tau(x,z)$. By using Lagrange polynomials, the resulting beam theories have only pure displacement unknowns. This class of theories have been called as Component-Wise (CW) in the literature (see [4,5]). In fact, as shown in Fig. 1, in the Component-Wise approach,finite beam elements based on Lagrange polynomial expansionscan be used to model the mechanical behaviour of the single component within the structures with high accuracy and absolute geometrical correctness.

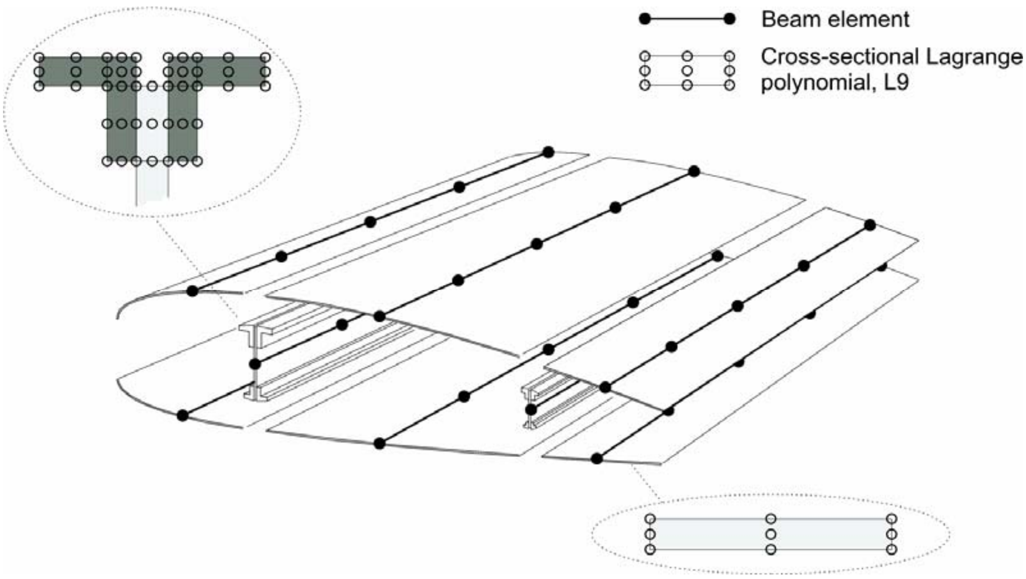

Fig.1. Component-wise modelling of a wing structure

The accuracy of the CW approach when applied to aerospace, civil engineering structures and marine constructions has been tested in the present work. Figure 2, for example, show some mode shapes of complex structures by the present 1D models.

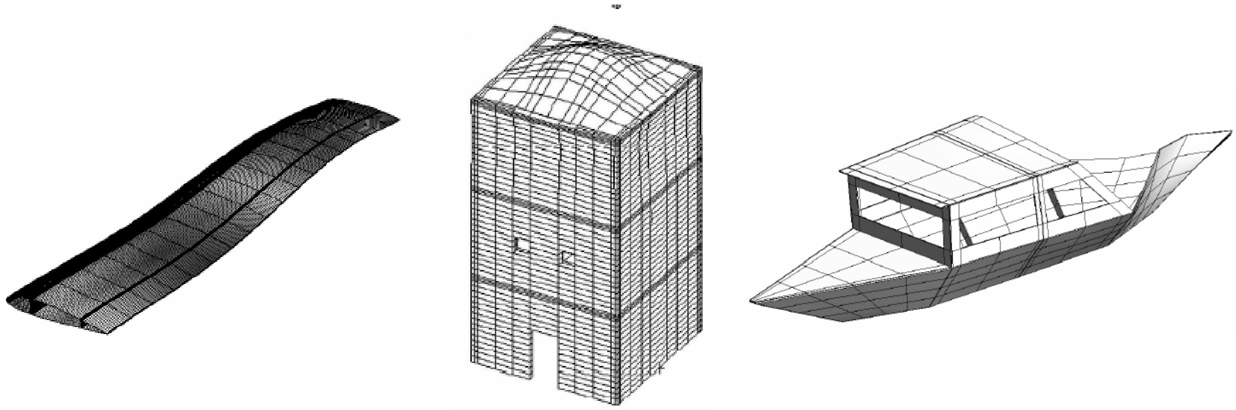

Fig. 2. Application of 1D CW models to the modal analysis of aerospace, civil engineering and marine constructions.

Two main aspects are clear from the analyses conducted:

- The CW approach, by exploiting Lagrange polynomials approximation on the beam cross-section, allows one to model multi-component structures without introducing any geometrical simplification.
- The CW models have the same accuracy of complex 3D FEM models with at least one order of magnitude less degrees of freedom.

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